Statistics and Data Analysis

Minimum Likelihood and Least Squares

1. Find the maximum likelihood estimators for the mean and the variance of a Gaussian distribution, and obtain the covariance matrix from the second derivatives of the likelihood.

2. Show that the maximum likelihood method applied to the x_i, y_i data points, where the y_i are distributed according to the double exponential, leads to the "least modulus" method (instead of the least squares).

3. Generate 2000 random numbers, $\{y_i\}$, following $f(x) = 1 + a_1\, x + a_2\, x^2$, using $a_1 = 0.5$, $a_2 = 0.5$, for $-1 \leq x \leq 1$. Use these data to perform a likelihood fit to estimate a_1, a_2. Draw the associated contour plots together with the real value and the estimate.

4. Find the parameters of $y = a_1 + a_2\, x + a_3\, x^2$ that best adjust the data $\{x_i, y_i \pm \sigma_i\}$: $(-0.6, 5 \pm 2); (-0.2, 3 \pm 1); (0.2, 5 \pm 1)$ and $(0.6, 8 \pm 2)$. Resolve the problem both "by hand" and using a computer.

5. An experiment measures the amplitude of a quantum process as $A = a_1 + i\, a_2$, with the result $a_1 = 0.12$ and $a_2 = -0.25$, with covariance matrix V_1. A second experiment measures only the real part of the amplitude, with $a_1 = 0.01 \pm 0.08$. Calculate the combined result and the error matrix.

$$V_1 = \begin{pmatrix} 0.01 & -0.01 \\ -0.01 & 0.04 \end{pmatrix}$$